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COMMENTS ON THE PAPER "BUBBLE GROWTH RATES AT HIGH JAKOB NUMBERS"

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I READ the paper on growth rates of adhering vapour bubbles in water and in some pure organic liquids boiling at sub-atmospheric pressures by Cole and Shulman [1] with great interest. A previous study on the nucleate boiling peak flux density had been carried out in this pressure range by Van Stralen [2,3] to water and to a number of aqueous binary mixtures. Extremely large vapour bubbles, generated at low frequences were observed at a pressure of 0·13 bar. a (10 cm Hg), cf. Fig. 38-2 of [3], but no systematic measurements of the bubble growth rates could be made, as a high-speed motion picture camera was not at my disposal at that time.

Some remarks may be useful for a more thorough interpretation of the experimental data presented in [1]. The nomenclature of [4,5] is adopted in this discussion.

1. EVALUATION OF THE EQUIVALENT BUBBLE VOLUME

The bubble diameter is defined as the diameter of a sphere having the same volume, and the bubbles are assumed to be ellipsoidal in shape [1]. Van Wijk and Van Stralen [6] and Van Stralen [7] adopted a similar procedure previously for *released* bubbles. However, these workers considered the *vertical* semi-axis a_2 (more generally for wires of circular cross-section: perpendicularly to the heating surface) as a rotation axis, especially in case of oscillating bubbles in the most important mode of vibration, the slowest fundamental harmonic. As a consequence, the equivalent bubble radius is then given by:

$$R = (a_1^2 a_2)^{\frac{1}{3}}. \qquad (1)$$

where a_1 denotes the horizontal semi-axis. In most cases the calculations could be simplified by taking:

$$S = \frac{2a_1 + a_2}{3}. \qquad (2)$$

For the bubbles investigated, $0{\cdot}8 \leqslant a_1/a_2 \leqslant 1{\cdot}2$ occurred generally. The values calculated from (2) were not more than 0·5 per cent (second-order deviation) too large in comparison with those following from (1). This can be verified easily by taking $a_2 = a_1 (1 \pm \Delta a)$, where $\Delta a = 0{\cdot}2$:

$$\frac{S}{R} = \left\{1 + \frac{(\Delta a)^2}{3(1 \pm \Delta a)}\right\}^{\frac{1}{3}} \approx 1 + \frac{(\Delta a)^2}{9}(1 \mp \Delta a) \approx 1{\cdot}005. \qquad (3)$$

Of course, more exact values of the bubble volume can be obtained by making use of the well known elementary Guldin mensuration rule, but the work is very elaborate in practice. Even then, rotation symmetry about any axis must be assumed, as motion pictures showing two or more simultaneous cross-sections of irregularly shaped (large) bubbles, which occur shortly before departure, are not available at present. Apparently, the tracings in Fig. 2 of [1] indicate symmetry about the vertical main axis also, and not about the horizontal axis, cf. Fig. 4 of part I of [4]. For *adhering* bubbles the shape of a spherical segment with radius R^* was assumed, cf. Section 2.3.2 of part I of [4]. The equivalent radius follows then from:

$$R = \{B^2(3 - 2B)\}^{\frac{1}{3}} R^*. \qquad (4)$$

2. COMPARISON OF EXPERIMENTAL DATA WITH THEORETICAL PREDICTIONS

Cole and Shulman [1] compare their experimental bubble growth curves with theoretical predictions following from the uniform superheating model (a), (especially for moderate Jakob numbers, which occur at moderate pressures), with Zuber's theory (b) and with a new empirical expression (especially for high Jakob numbers occurring at low pressures), cf. Section 5. We shall now also make a comparison with Van Stralen's "relaxation microlayer" theory (c), including the behaviour of bubbles in binary mixtures, cf. Sections 3 and 4.

(a) Equation (12) of [1] can be written:

$$R = 0{\cdot}50\, Ja\, (\pi a t)^{\frac{1}{2}}, \qquad (5)$$

i.e. the Forster–Zuber equation (multiplied by a factor of 0·50) for bubble growth in an initially uniformly superheated pure liquid of infinite volume. The factor of 0·50 had been included as the Jakob number $Ja = (\rho_1 c/\rho_2 l)\theta_0$ is related to the (average) superheating θ_0 of the heating surface, which determines the rapid initial growth. Cole and Shulman's data are in good agreement with the modified uniform model (5) for $Ja < 100$. Contrarily, for $Ja = 800$, equation (5) predicts bubble growth coefficients, "which exceed the experimental values by an order of magnitude". It is convenient to introduce an average Jakob number

$\overline{Ja} = 0{\cdot}50\ Ja$, which simplifies the right-hand side of equation (5). It may be worth noticing here, that Scriven's additional term $(1 - c_1/c_2)\theta_0$ in the denominator of equation (11) of [1] must be omitted, as (within a few milliseconds) vaporization at the bubble boundary occurs actually at the saturation temperature.

Cole and Shulman's experimental constant value of the dimensionless growth coefficient β in Scriven's equation for the bubble radius:

$$R = 2\beta(at)^{\frac{1}{2}} \tag{6}$$

is surprising, because generally $\beta = \beta(t)$ is proportional to $\theta(t)$, the instantaneous liquid superheating of the surrounding "relaxation microlayer". Generally, for adhering bubbles, one expects to observe very much higher initial values of β (because the equilibrium radius R_0 is an order of magnitude smaller than $d_{w,co}$) in comparison with the value at the instant of breaking away from the heating surface, viz., the relatively slow bubble growth rate at departure is only due to the small superheating $\Delta\theta_0$ (0·1–1·0 degC) of the bulk liquid outside the equivalent thermal conduction layer at the heating surface. This prediction is in good agreement with Van Stralen's [4–7] experimental data on horizontal platinum wires.

Unfortunately, Cole and Shulman give no information on the recorded local liquid temperatures close to their zirconium heating ribbon. Possibly, the local $\Delta\theta_0$ has reached considerably higher values in their experiments in consequence of the placing of a glass plate below the ribbon. This may explain the time-independence of β, but contradicts the proposed factor of 0·50 in the right-hand side of equation (5). Perhaps their relatively slow initial growth rates are due to a decreased superheating of the upper surface of the ribbon in comparison with the average value, which is measured by resistance thermometry.

(b) A comparison is made with Zuber's [8] expression for bubble growth in a non-uniform temperature field, which is of special importance in case of surface boiling of subcooled liquids:

$$R = Ja(\pi at)^{\frac{1}{2}} \left\{1 - \frac{q_w(\pi at)^{\frac{1}{2}}}{2k\theta_0}\right\} \leqslant Ja(\pi at)^{\frac{1}{2}}. \tag{7}$$

According to the theoretical treatment by the reviewer, Zuber's assumption $q_w = q_b$ in equation (7) is fundamentally incorrect, cf. also Forster's [9] argumentation. In terms of the relaxation microlayer theory [4]:

$$\mathrm{d}_{0,}\, p = 0{\cdot}79\, d_{w,co} = 0{\cdot}79\, k\theta_0/q_{w,co} = (\pi at_1)^{\frac{1}{2}}, \tag{8}$$

if Plesset and Zwick's and Scriven's numerical constant $(12/\pi)^{\frac{1}{2}}$ is replaced by Forster and Zuber's value $\pi^{\frac{1}{2}}$. Substitution of (8) in (7) yields:

$$R = Ja(\pi at)^{\frac{1}{2}} \left\{1 - 0{\cdot}39\left(\frac{t}{t_1}\right)^{\frac{1}{2}}\right\}, \tag{9}$$

whence $R = Ja\,(\pi at)^{\frac{1}{2}}$ as $t \to 0$ and $R_1 = 0{\cdot}61\ Ja\,(\pi at_1)^{\frac{1}{2}}$ as $t = t_1$. The initial values of the bubble growth coefficient β for adhering bubbles are thus equal to those following from the uniform superheating model. During the entire adherence time, equation (9) predicts 2·0–1·2 times higher values of β in comparison with equation (5). This is in agreement with the theoretical curves in Figs. 6–23 of [1]. The reviewer is aware, that objections may be raised to the substitution of his equation (8) in Zuber's expression (7) by taking $q_w = q_{w,co}$.

(c) The discrepancy between the lower experimental values of β for high Jakob numbers in comparison with those following from equations (5) and (9) is avoided by the reviewers relaxation microlayer theory [4]. The equivalent bubble radius during adherence is given by the expression:

$$R(t) = bC_1\theta(t)\, t^{\frac{1}{2}} = bC_1\theta_0 \exp - \left(\frac{t}{t_1}\right)^{\frac{1}{2}} t^{\frac{1}{2}}. \tag{10}$$

The occurrence of the factor b, the dimensionless growth parameter, in equation (10) can be explained both in terms of a partial heat supply to the bubble boundary and in terms of a reduced average initial superheating $b\theta_0$ of the relaxation microlayer [10]. The departure radius follows from (10):

$$R(t_1) = R_1 = \frac{b}{e} C_1\theta_0 t_1^{\frac{1}{2}} = 0{\cdot}26\, C_1\theta_0 t_1^{\frac{1}{2}}, \tag{11}$$

or for pure liquids [4, 5], using the Plesset–Zwick and Scriven derivation for $C_{1,p}$:

$$R_1 = 0{\cdot}26\left(\frac{12}{\pi}\right)^{\frac{1}{2}} \frac{\rho_1 c}{\rho_2 l}\,\theta_0(at_1)^{\frac{1}{2}} = 0{\cdot}26\left(\frac{12}{\pi}\right)^{\frac{1}{2}} Ja(at_1)^{\frac{1}{2}}$$

$$= 2\beta_1(at_1)^{\frac{1}{2}}, \tag{12}$$

whence

$$\beta_1 = 0{\cdot}25\, Ja. \tag{13}$$

3. SURVEY OF EXPERIMENTAL DATA

In Fig. 1, the bubble growth coefficient for adhering bubbles ($Ja_0 = 1 - 10^3$) according to Van Stralen's theory [4] (with $b = 0{\cdot}72$), i.e. equation (13), is compared with experimental data of β by Cole and Shulman (○) for adhering bubbles in water, methanol, pentane and toluene (0·06–1·03 bar.a), cf. Fig. 5 of [1]; of β_1 by Van Stralen (●) for adhering bubbles at the instant of departure in water, (0·13–1·03 bar.a), cf. [2, 3, 6, 7], in 4·1 wt % methylethylketone in water ($C_{1,m} = 0{\cdot}25\ C_{1,p}$), (1·03 bar.a), and in 1·5 wt % 1-butanol in water ($C_{1,m} = 0{\cdot}75\ C_{1,m}$), (1·03 bar.a); and of β_1 by Séméria (■) for adhering bubbles at the instant of departure in water, (100 bar.a), cf. [11].

Also, in Fig. 1, a comparison is given for free or released bubbles ($Ja_0 = 0{\cdot}10$–10) with the uniform superheating model: data of β by Dergarabedian (△) for free bubbles in water, (1·03 bar.a), cf. [12]; by Van Stralen (□) for released bubbles in water, in 4·1 % methylethylketone, and in 1·5 % and 6·0 % ($C_{1,m} = 0{\cdot}87\ C_{1,p}$) 1-butanol, (1·03 bar.a), cf. [6, 7]; and by Benjamin and Westwater (▽) for adhering and released bubbles in water, (1·03 bar.a), cf. [13]. In pure

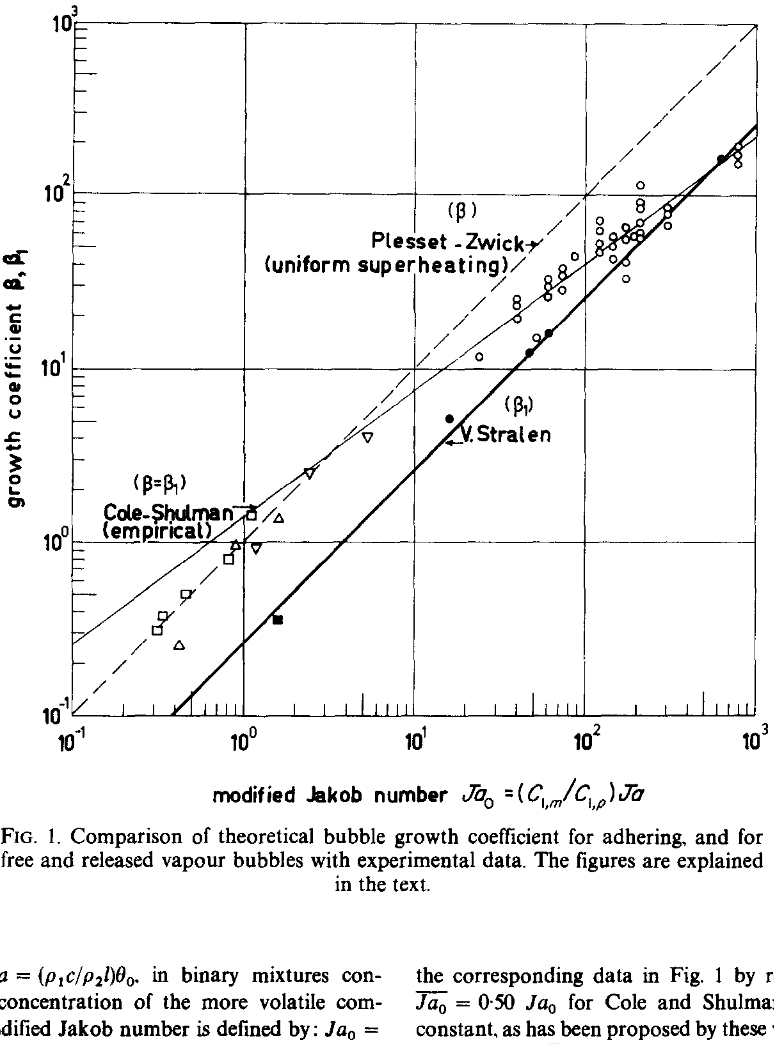

FIG. 1. Comparison of theoretical bubble growth coefficient for adhering, and for free and released vapour bubbles with experimental data. The figures are explained in the text.

liquids: $Ja_0 = Ja = (\rho_1 c/\rho_2 l)\theta_0$, in binary mixtures containing a small concentration of the more volatile component a new modified Jakob number is defined by: $Ja_0 = (C_{1,m}/C_{1,p})Ja = (C_{1,m}/C_{1,p})(\rho_1 c/\rho_2 l)\theta_0$, cf. [5]. In general, low (modified) Jakob numbers can be obtained in three ways: (i) at high pressures (Séméria), as $C_{1,p} \sim 1/\rho_2 \sim 1/p$; (ii) at low superheatings $\Delta\theta_0$ of the bulk liquid (Dergarabedian, Van Stralen); (iii) in binary systems at the concentration of minimal bubble growth constant $C_{1,m}$ (Van Stralen). High Jakob numbers occur for adhering bubbles in nucleate boiling of pure liquids at low pressures (Cole and Shulman), and are also reached in filmboiling due to the large effective θ_0 at the vapour–liquid interface close to the heating surface (Van Stralen, cf. [5]).

4. MODIFICATION FOR ADHERING BUBBLES TO A GENERAL EQUATION

Evidently, in Fig. 2, the data for free and released bubbles are unaltered in comparison with Fig. 1. For bubbles growing at a heating surface, the data in Fig. 2 follow from the corresponding data in Fig. 1 by replacing: (i) Ja_0 by $\overline{Ja_0} = 0{\cdot}50\ Ja_0$ for Cole and Shulman's data, keeping β constant, as has been proposed by these workers, cf. equation (5); (ii) Ja_0 by $\overline{Ja_0}$, and, simultaneously, β_1 at the instant of departure by $\overline{\beta(t)}$, averaged over the entire adherence time, for Van Stralen's and Séméria's data.

In the latter case: $\overline{\theta(t)} = 0{\cdot}53\ \theta_0$, cf. equation (10) and equation (50) of part I of [4], whence $\overline{Ja_0} = 0{\cdot}53\ Ja_0$. This factor differs only slightly from 0·50. Analogously, $\overline{\beta(t)} = (\pi/3)^{\frac{1}{2}} b\ \overline{Ja_0} = 0{\cdot}70\ \overline{Ja_0} = 0{\cdot}37\ Ja_0$, whence

$$\bar{\beta} = 0{\cdot}70\ \overline{Ja_0}. \tag{14}$$

Equation (14) is in good agreement with all data for adhering bubbles (Fig. 2).

5. COMPARISON WITH COLE AND SHULMAN'S EMPIRICAL RELATION: $R \sim Ja^{0{\cdot}75}(at)^{\frac{1}{2}}$

One can derive from equation (10) an expression for

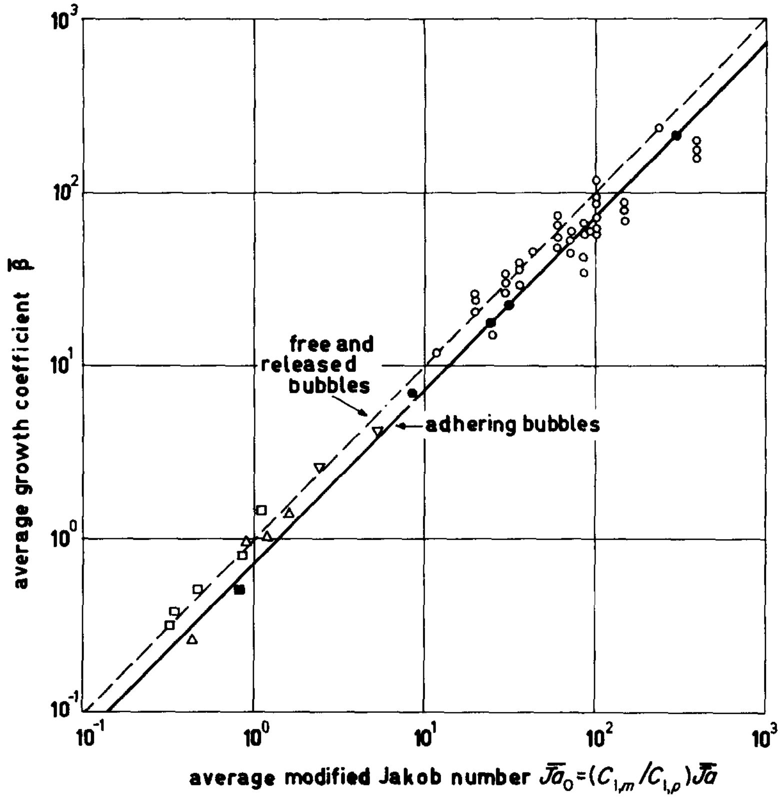

FIG. 2. Extension of data in Fig. 1 to corresponding average modified Jakob numbers. Same figures in Figs. 1 and 2 refer to same experimental data.
The curve and data for adhering bubbles tally with the uniform superheating model, if $\overline{Ja_0}$ is replaced by $b\overline{Ja_0}$, as this has no effect on $\bar{\beta}$.

$\overline{R(t)}$, averaged over the entire adherence time:

$$\overline{R(t)} = 2\left(2 - \frac{5}{e}\right) bC_1\theta_0 t_1{}^{\frac{1}{2}} = 0{\cdot}32\, bC_1\theta_0 t_1{}^{\frac{1}{2}}. \qquad (15)$$

One has for pure liquids, according to equation (8):

$$(\pi a t_1)^{\frac{1}{2}} = 1{\cdot}27\, d_{w,co} = 1{\cdot}27\, k\theta_0/q_{w,co} \sim \theta_0/\theta_0^{1\cdot 25} \sim \theta_0^{-0\cdot 25}. \qquad (16)$$

since the convective contribution to the heat flux density of heating wires with the same diameter is proportional to $\theta_0^{1\cdot 25}$. Substitution of (16) in (15) yields: $\overline{R(t)} \sim \theta_0^{0\cdot 75} \sim \overline{Ja_0}^{0\cdot 75}$. The exponent of $\overline{Ja_0}$ is the same as in Cole and Shulman's equation. However, one has to consider, that the factor $t^{\frac{1}{2}}$ is eliminated now; this is not the case for $\overline{\beta(t)}$, but equation (14) is then giving an exponent 1·00 for $\overline{Ja_0}$. The discrepancy originates from the time-dependence of $\beta(t)$ in contradistinction to Cole and Shulman's constant values.

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